On Designing Protocols for Non-cooperative Multi-Radio Channel Assignment in Multiple Collision Domains

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***Abstract*—Channel assignment is a crucial problem for wireless networks, especially for non-cooperative wireless networks, in which nodes are selfish. While there have been a few studies of non-cooperative, multi-radio channel assignment, most existing studies are restricted to single collision domains only. In this paper, we study the design of incentive-compatible protocols for non-cooperative, multi-radio channel assignment in *multiple collision domains*.**

First, we show the necessity of designing incentive-compatible protocols for this problem. Specifically, we show that, if no incentive-compatible protocol is deployed, Nash Equilibria (NEs) may have undesired properties, such as Pareto suboptimality and low throughput.

In order to prevent the system from converging to the NEs with undesired properties, we propose an incentive-compatible protocol for channel assignment in multiple collision domains. We rigorously show that our protocol guarantees that the system converges to NEs that are Pareto-optimal and have the maximum system-wide throughput. Our simulation results also verify that our protocols are effective in ensuring that the system converges to the desired NEs.

***Index Terms*—Wireless Access, Channel Assignment, Mechanism Design.**

I. Introduction

Frequency Division Multiplexing Access (FDMA) is a frequently used multiplexing technique in wireless networks. FDMA divides the carrier bandwidth into a number of sub-bands, called channels. The wireless devices need to assign their radio transmitters to these channels, so that they can transfer signals simultaneously. This classical problem of channel assignment is of great importance to wireless communications and thus has been studied extensively [1]- [7]. In particular, when the involved mobile devices have multiple interfaces, this problem becomes the *multi-radio channel assignment*, which has been addressed in some existing works (e.g., [5]–[7]).

Recently, a new variant of the multi-radio channel assignment problem, *non-cooperative, multi-radio channel assignment* [8], has attracted a lot of attention. When wireless devices are non-cooperative (i.e., *selfish*), traditional channel assignment protocols, which have been designed for cooperative devices, can no longer be used. The reason is that selfish devices may deviate from the protocols for their own benefits.

While a number of interesting results have been obtained on non-cooperative, multi-radio channel assignment, existing studies are restricted to single collision domains only. For example, Felegyhazi *et al.* [8] are the first to study non-cooperative, multi-radio channel assignment in a single collision domain. They assume that the involved wireless devices are all within a single hop from each other. Wu *et al.* [9], [10] work in a similar setting and design a channel assignment protocol that can achieve globally optimal throughput. Gao and Wang [11] remove the single hop assumption and obtain very nice results by modeling the multiple hop channel allocation problem as a static cooperative game. We note that removing the assumption of single hops is *not* identical to removing the assumption of single collision domain, because of the difference between transmission range and sensing range. In particular, in [11], Gao and Wang still keep assumption that players reside in a single collision domain.

In this paper, we systematically study the problem of non-cooperative, multi-radio channel assignment in *multiple collision domains*. Our ultimate goal is to design *incentive-compatible channel assignment protocols* in this setting, such that even in the presence of selfish devices, the network system can still converge to stable states with desirable properties, such as high system-wide throughput and Pareto optimality.

In order to design the incentive-compatible proto-

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col, we first show its *necessity* by the game theoretic analysis. Specifically, we investigate the possible stable states, namely *Nash Equilibria* (NEs), that the system could converge to, if no incentive-compatible channel assignment protocol is deployed. (In practice, the system should evolve to one of the NEs and then permanently stay in that state.) We obtain quantified results on the economic efficiency and throughput of these NEs. Our results indicate that these NEs may have undesired properties. For example, some NEs can be Pareto-suboptimal, which means that there are better states of the system giving more payoffs to some devices than these NEs without decreasing other devices' payoffs. Hence, if the system finally evolves to one of such NEs, then some devices lose part of their payoffs unnecessarily. Moreover, we show that some NEs may have low system-wide throughputs.

To prevent the system from converging to these NEs having undesired properties, we propose an incentive-compatible channel assignment protocol for multiple collision domains. This protocol guarantees the Pareto optimality of all NEs and maximizes the system-wide throughput of them. The main tool we use to build this protocol is payment—we require a user to pay an amount of virtual currency for her devices' use of the channels. We argue that this is a *natural* requirement since communication bandwidth is a type of resource and it is reasonable to request the users to pay for their usage of resources. Furthermore, these payments can be collected in a *secure* and *efficient* way, and may *not* require an online central authority, as discussed in [13]–[15].

In summary, we make the following contributions in this paper:

- We study the problem of non-cooperative, multi-radio channel assignment in multiple collision domains, using a mechanism design approach.
- We analyze the NEs of the multi-radio channel assignment game in multiple collision domains and obtain quantified results on economic efficiency and throughput. Our results indicate that designing incentive-compatible protocols is necessary, because otherwise the system may converge to a NE that is Pareto-suboptimal or has low system-wide throughput.
- To guarantee that stable states (i.e., NEs) of the system always have the desired properties, we propose an incentive-compatible channel assignment protocol for non-cooperative, multi-radio channel assignment. This protocol guarantees that the NEs maximize the system-wide throughput, and that all the NEs are Pareto-optimal. We show the properties of this protocol with rigorous analysis.
- We perform extensive evaluations on GloMoSim [16]. The results show that our protocol is effective in ensuring that the system converges to the desired states.

The rest of this paper is organized as follows. First, we introduce the technical preliminaries and our game model for multi-radio channel assignment in multiple collision domains in Section II. In Section III we motivate the need for incentive-compatible channel assignment protocols by analyzing the properties of Nash equilibria in this game. Then in view of the undesired properties of NEs, we propose an incentive-compatible protocol to maximize the system throughput and achieve Pareto optimality in Section IV. We present the evaluation results in Section V. Finally, after briefly reviewing the related work in Section VI, we conclude our paper in Section VII.

II. Preliminaries

In this section, we first present our system model, then describe the channel assignment game that we study, and finally review the definitions we use in this paper.

A. System model

In our model, we assume a network that consists of a number of node pairs. Let P denote the set of node pairs in the network. For the entire network the available frequency band is divided into orthogonal channels (e.g., 8 orthogonal channels in IEEE 802.11a protocol), the set of which is denoted by C. The channels are assumed to have the same characteristics. Each node has K transceivers to use. We assume that the MAC layer coordination function is turned off. The two nodes in each pair are within the transmission range of each other. They can establish a bidirectional communication, by tuning a pair of transceivers (one transceiver from each node) to the same channel. There is a mechanism that enables each node pair to simultaneously transmit packets using multiple channels. Each node is only involved in one such node pair.

We consider multiple collision domains. That is, some node pairs cannot interfere with the communications of some other pairs, even if they are all using the same channel. Two node pairs can interfere with each other's communication only when they are within the *interference range* of each other.

B. Multi-radio channel assignment game in multiple collision domains

In this paper, our goal is to design incentive-compatible channel assignment protocols for multiple collision domains, to achieve desirable system properties. Here by incentive-compatible, we mean that even though each node in the system can control his radios, it is still to his best interest to assign his radios to channels in a way such that desirable system performance can be achieved. To provide incentives to each node, we design suitable payments for the channel usage. This can be viewed as an application of mechanism design to the wireless network channel assignment problem in multiple collision domains. For a general introduction to the mechanism design literature, please refer to [17]. In this paper we take game-theoretic approach to mechanism design.

We model the multi-radio channel assignment problem in multiple collision domains as a non-cooperative strategic game, in which each pair of communicating nodes is a selfish player. The set of players is thus P. The objective of each player is to maximize its own communication throughput and to minimize the cost at the same time. Note that the attempt to transmit packets may not be successful due to interference. We use the interference model (e.g. in [18]) that if two players within each other's interference range are transmitting packets on the same channel at the same time, no one can successfully transmit any useful data. Under this interference model, each player will not put more than one radio on the same channel at the same time, to avoid the interference with himself.

Each player's strategy in the game is to decide whether to use its radios and which channels to put radios on.

Formally, the strategy of player i is defined as

$$s_i = \{S_i^c | c \in C\},$$

where

$$S_i^c = \begin{cases} 1 & \text{if player } i \text{ has one radio on channel c} \\ 0 & \text{if player } i \text{ has no radio on channel c} \end{cases}$$

Since each player only has K radios, the number of channels used by player i (denoted by k_i), can not exceed K. (i.e. $\forall i, k_i = \sum_{c \in C} S_i^c \le K$). The strategy profile is a matrix composed of all players' strategies, $s = (s_1, s_2, \cdots, s_{|P|})$. The strategy profile except for i's strategy is denoted as s_{-i}.

Whether players can successfully transmit packets depends on their strategies as well as those of others which may cause interference to them. We use flow contention graph[1] to illustrate the interference relationship between players. In the flow contention graph, each node represents a player. If and only if two players are within each other's interference range, there is an edge between the two nodes in the flow contention graph. Fig. 1 shows an example of flow contention graph. The topic of how to obtain the flow contention graph is closely related to the wireless network topology discovery problem which has been well studied (e.g., [22], [23]). We can adopt some of the available adaptive topology discovery algorithms (e.g., [24]), but since the topic of topology discovery is already beyond the scope of this paper, we will not explore it in detail.

For player i, the set of players who are connected with i (including i itself) in the flow contention graph is called i's interference set, denoted by N_i. We also define $n_{max} = \max_{i \in P} |N_i|$.

[1]All flows are single-hop flows in our game and each node in flow contention graph represents a player or his flow.

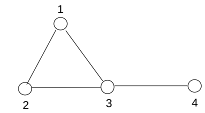

Fig. 1. An example of flow contention graph.

TABLE I
TABLE OF NOTATIONS

P	Set of players
s_i	Strategy of player i
K	The number of radios that each player has
S_i^c	The number of radio that player i has on channel c
k_i	The number of channels used by player i
s	Strategy profile of all players
s_{-i}	The strategy profile except for i's strategy
N_i	Player i's interference set in the flow contention graph
n_{max}	The size of the largest interference set
r	the amount of data that a player can transmit through one radio
β	Energy cost parameter
u_i	Player i's utility

Now we define the payoff function of player i as the amount of data that i successfully transmits, minus the cost of transmission. Formally,

$$u_i = \sum \left(r \sum_{c \in C} \left(S_i^c \cdot \prod_{j \in N_i} (1 - S_j^c) \right) - \beta \cdot k_i \right), \tag{1}$$

in which r is the throughput that a player can obtain through one radio, and $\beta (\beta < r)$ is a constant number representing the energy cost rate for one radio. Given the interference model described above, we know that each player can perform successful transmissions on one channel only when all the players in his interference set do not use that channel. If any of the players in the interference set is attempting to transmit, there will be collisions. Correspondingly, in Eq. (1), if none of the neighbor players of i use channel c, then $\prod_{j \in N_i}(1 - S_j^c) = 1$. $\prod_{j \in N_i}(1 - S_j^c) = 0$ implies that at least one neighbor player of i has a radio on channel c. In this case, even if i puts one radio on channel c (i.e., S_i^c=1), he will not successfully transmit data and as a result he will lose the corresponding share of payoff.[2] We summarize the important notations used in this paper in Table II-B.

[2]We note that there could be some DoS attackers who are willing to sacrifice payoff initially by jamming other users until some of them drop out. We assume that this type of DoS attacks can be detected by the network administrators and once detected, the attackers will be removed away from the network service.

C. Definitions

To analyze the channel assignment game, we use some of the definitions (as described below) from game theory. For completeness, we include these definitions below. (Readers interested in these definitions can refer to, e.g., [25] for detailed discussions.)

Definition 1: (Nash Equilibrium (NE)) Let (S, U) be a game with the player set P, where s_i is the strategy set for player i, $S = s_1 \times s_2 \times \cdots \times s_{|P|}$ is the set of strategy profiles, and $U = (u_1(s), u_2(s), \cdots, u_{|P|}(s))$ is the utility functions for $s \in S$. The strategy profile $s^* = \{s_1^*, s_2^*, \cdots, s_{|P|}^*\}$ is a Nash equilibrium (NE) if for every player $i \in P$, we have

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \qquad (2)$$

for all strategy s_i.

NEs are the *stable states* of the system, because no single player has incentives to leave them. Normally, the system should converge to a NE and then permanently stay there. Consequently, it is important to guarantee that NEs have good properties such as economic efficiency. A definition often used for economic efficiency is Pareto optimality:

Definition 2: (Pareto Optimality) Let (S, U) be a game with the player set P, where s_i is the strategy set for player i, $S = s_1 \times s_2 \times \cdots \times s_{|P|}$ is the set of strategy profiles, and $U = (u_1(s), u_2(s), \cdots, u_{|P|}(s))$ is the utility functions for $s \in S$. A strategy profile s^{po} is Pareto-optimal if for every strategy profile s such that there exists player $i \in P$,

$$u_i(s^{po}) < u_i(s),$$

there must exist another player $j \in P$,

$$u_j(s^{po}) > u_j(s).$$

Intuitively, in a Pareto-optimal state, no player can get more payoff without hurting another player. Clearly, it is desirable to guarantee that all NEs are Pareto-optimal.

III. Necessity of Designing Incentive-Compatible Protocols

In this section, we show the necessity of designing incentive-compatible protocols for non-cooperative, multi-radio channel assignment. In particular, we rigorously analyze the NEs in a system without incentive-compatible protocols, and study their economic efficiency and throughput.

Before we analyze the properties of NEs in the channel assignment game, we first characterize them by providing a necessary and sufficient condition for strategy profiles to become NEs.

Theorem 1: s^* is a NE if and only if the following two conditions hold:

(1) $\forall i, \forall c, \text{if } S_i^{c*} = 1, \text{then} \prod_{j \in N_i}(1 - S_j^{c*}) = 1$

(2) in any channel c, there does not exist player i, s.t.

$$\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0, \text{and } k_i^* < K.$$

We first introduce two Lemmas to help the proof of Theorem 1.

Lemma 1: If s^* is a NE, then

$$\forall i, c, \text{if } S_i^{c*} = 1, \ \prod_{j \in N_i}(1 - S_j^{c*}) = 1.$$

Proof: We prove this by contradiction. Suppose $\exists i, c \ s.t. S_i^{c*} = 1$ and $\prod_{j \in N_i}(1 - S_j^{c*}) = 0$. We consider another strategy for i, s_i', which equals s^* except for $S_i^{c\prime} = 0$. Then we compare the utilities of player i taking strategy s_i^* and s_i' when the strategies of players remain the same.

$$\begin{aligned} u_i' - u_i^* &= S_i^{c\prime} \prod_{j \in N_i}(1 - S_j^{c*})r - \beta S_i^{c\prime} \\ &\quad -(S_i^{c*} \prod_{j \in N_i}(1 - S_j^{c*})r - \beta S_i^{c*}) \\ &= 0 - (-\beta) \\ &> 0 \end{aligned}$$

This contradicts with the fact that s^* is a Nash equilibrium. ■

Another straightforward necessary condition of NEs is that players will put as many radios as possible on channels to increase their utilities as long as there is no interference with others. Formally we have Lemma 2.

Lemma 2: If s^* is a NE, then there does not exist i, s.t.

$$\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0, \text{and } k_i < K.$$

Proof of Theorem 1

Proof: Since we already have Lemma 1 and Lemma 2, all we need to prove here is that if the two conditions hold, s^* is a NE.

Suppose that under the two conditions above, a player i can unilaterally increase his utility by changing his strategy to u_i'. He has two possible ways in total to do so.

- Changing some S_i^{c*} from 1 to 0.

If $S_i^{c*} = 1$, from condition (1), we know that $\prod_{j \in N_i}(1 - S_j^{c*}) = 1$. In this case $u_i' - u_i^* \leq 0$. Therefore, by changing some S_i^{c*} from 1 to 0, i can not increase his utility.

- Changing some S_i^{c*} from 0 to 1.

We now consider two cases.

If $\sum_{j \in N_i} S_j^{c*} + S_i^{c*} > 0$, then $\prod_{j \in N_i}(1 - S_j^{c*}) = 0$. In this case, if i changes S_i^{c*} to 1, it will decrease his utility by β.

If $\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0$, from condition (2) we know that, it must be the case that $k_i = K$, which means i has no spare radios to improve his utility.

Therefore there is no way for i to unilaterally increase his utility with others strategies being equal. Hence, s^* is a Nash equilibrium. ■

Condition (1) suggests that players will avoid interference to maximize their payoffs. Condition (2) says no player wants to spare their radios if they could successfully transmit packets. If both (1) and (2) are satisfied, the system is in its NE and vice versa. If in the system each node always tries to change his channel assignment for better utility in a distributed fashion, the system will converge to NE status as described in Theorem 1. This is due to the definition of NE and that the status in Theorem 1 is within the system capacity. Although the system will always converge, it is still non-trivial to determine whether these NEs can guarantee desired system properties. Hereafter we will use Theorem 1 in the analysis of NEs' properties.

A. Economic Efficiency

In this subsection, we study the property of NEs from a system-wide perspective, *economic efficiency*[3], using Pareto optimality as the criterion. If the system converges to a NE that is not Pareto-optimal, then some players lose the opportunities of increasing their own payoffs without hurting anyone else, which immediately implies that some resources in the system are wasted. Therefore, it is important to identify whether all NEs in the channel assignment game are Pareto-optimal.

First we observe an example.

Example 1. Consider a network with three players and the flow contention graph is shown as Fig. 1. Each player has 2 radios and there are 3 channels, a, b, c, available.

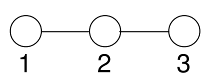

Fig. 2. The flow contention graph in Example of Pareto suboptimality.

Consider a strategy profile $s = \{s_1, s_2, s_3\}$, $s_1 = \{\forall t, S_1^a = 1,\ S_1^b = 0, S_1^c = 1\}$, $s_2 = \{\forall t, S_2^a = 0,\ S_2^b = 0,\ \ S_2^c = 0\}$, $s_3 = \{\forall t, S_3^a = 0, S_3^b = 1,\ S_3^c = 1\}$. In words, player 1 is using channel a and c; player 3 is using channel b and c; player 2 has no radio in use. Here s achieves a Nash equilibrium, because player 1 and 3 both have obtained their best possible payoffs and player 2 has no way to improve his payoff given the fact that no matter which channel (a or b) he tries to use there will be an interference. However s is not Pareto-optimal. In fact, if player 1 moves one of his radios from channel a to channel b, player 2 can start using one of his radios to transmit packets on channel a without any interference. In this way player 2 increases his payoff without decreasing any other player's payoff, which implies that s is Pareto-suboptimal.

This example shows that NEs may not be Pareto-optimal in the non-cooperative, multi-radio channel assignment. But what are the exact conditions for NEs to be Pareto-optimal or Pareto-suboptimal? Is there a system that has all its NEs being Pareto-optimal?

Our main observation is that Pareto optimality depends on the values of K, $|C|$ and n_{max}. More precisely, Pareto optimality can be guaranteed in all NEs when $|C|$ is not less than $n_{max} \cdot K$ (Proposition 1), or not more than K (Proposition 2). If the value of $|C|$ is between these two thresholds, there can be some NEs that are Pareto-suboptimal, as we have shown in Example 1.

Proposition 1: If $|C| \geq n_{max} \cdot K$, all the Nash equilibria are Pareto-optimal [4].

Proof: First we show that if $|C| \geq n_{max} \cdot K$, in any Nash equilibrium, $\forall i,\ k_i = K$. That is, each player is using all his radios to transmit packets. Recall that n_{max} is the maximum number of players that can interfere with one player. Since $|C| \geq n_{max} \cdot K$, for each player, no matter what strategies other players take, there are always more than K idle channels to put his radios on without interference. If a player is using less than K radios, it contradicts with the definition of NE.When all the radios of players are occupied, there is no way to increase the payoff of any player. Therefore, the Nash equilibrium strategy profiles are Pareto-optimal. ■

Proposition 2: If $|C| \leq K$, all the Nash equilibria are Pareto-optimal. [5]

Proof: If $|C| \leq K$, for any Nash equilibrium, suppose it is not Pareto-optimal, i.e. there is a player i that can increase his utility without decreasing any other player's utility.

First we have an observation. In a Nash equilibrium, a necessary condition for i to increase his utility by changing his strategy from s_i^* to s_i' is that

$$\exists (c,t),\ s.t.\ S_i^{c*} = 0 \text{ and } S_i^{c\prime} \cdot \prod_{j \in N_i} (1 - S_j^{c\prime}) = 1.$$

Since $S_i^{c*} = 0$, we can get $\sum_{j \in N_i} S_j^{c*} \geq 1$. Then $\exists k \in N_i,\ s.t.\ S_k^{c*} = 1$. In order to have player k not decreasing his utility after i puts a radio on c, there must be at least one channel c' for player k to move his radio to. Formally, $S_k^{c'*} = 0$ and $S_k^{c''} \prod_{j \in N_k} (1 - S_j^{c''}) = 1$.

Now we consider two cases regarding to $\prod_{j \in N_k} (1 - S_j^{c'*})$, to show that no matter in which case there will be a contradiction.

Case 1. If $\prod_{j \in N_k} (1 - S_j^{c'*}) = 1$, then $\forall j \in N_k, S_j^{c'*} = 0$. Hence we have $\sum_{j \in N_k} S_j^{c'*} + S_k^{c'*} = 0$, which is contradicting with Theorem 1.

[3]Note that economic efficiency is a standard term for resource allocation in economic theory, even though in many cases real money is not involved.

[4]The bound for $|C|$ is tight, i.e., this proposition holds when $|C| = n_{max} \cdot K$.

[5]The bound for $|C|$ is tight in this proposition. Please see the proof for details.

Case 2. If $\prod_{j \in N_k}(1 - S_j^{c'^*}) = 0$, then $\exists j' \in N_k, s.t. S_{j',t}^{c'} = 1$. Now player j' meets the same situation with player k that j' must move his radio on channel c' to another channel to keep his utility from decreasing. Note that the process of having one player switch his radio to another channel must stop to achieve a successful Pareto improvement, while the process can only stop in Case 1 for some player, which introduces contradiction.

Therefore, if $|C| \leq K$, all Nash equilibria are Pareto-optimal.

■

An intuitive explanation of Proposition 2 is that when $|C| \leq K$, since the channel resource is so limited, in a Nash equilibrium if a player wants to increase its payoff by employing one more radio in some channel, at least one of its neighbors must remove its radio from that channel. Because in a Nash equilibrium, for each player, there is no more available channel to use, the change that a player uses one more channel must result in the consequence that some other player loses part of its utility due to the decreased number of occupied channels.

The above two propositions tell us that if the number of channels available is large enough ($|C| \geq n_{max} \cdot K$) or small enough ($|C| \leq K$), any NE channel allocation is Pareto-optimal. It implies that in these two cases, the system administrators do not have to consider economic efficiency when choosing channel assignment protocols and thus can focus on other properties such as throughput. But note that considering the current real applications, both $|C| \geq n_{max} \cdot K$ and $|C| \leq K$ are minor cases.

Now we study the remaining cases, in which $K < |C| < n_{max} \cdot K$. Let us revisit Example 1 in which NEs are not Pareto-optimal. In Example 1, $K = 2$, $|C| = 3$, $n_{max} = 3$. (The size of interference set of Node 2 (containing Node 2 itself) is 3.) We have $K < |C| < n_{max} \cdot K$. So if $K < |C| < n_{max} \cdot K$, there may be some Nash equilibria which are not Pareto-optimal.

B. Throughput

The second property of NEs that we study is system-wide throughput. Let $I(i)$ denote the interference degree of i—the number of players in the interference set of i that can transmit packets simultaneously without interfering with each other. Let $I(G)$ denote the maximum interference degree among all the players. Let r^* denote the maximum system-wide throughput that a network can achieve. In fact, the system-wide throughputs of some Nash equilibria can be as low as $r^*/I(G)$. Below we give an example of low throughput NEs.

Example 2. Consider a network with the flow contention graph shown in Fig. 3, where $|C| = 2$ and $K = 2$. Clearly, the maximum system-wide throughput is achieved when players 1 through n use the two channels. However, the system could converge to a Nash equilibrium, in which only player 0 transmits packets using 2 radios. In this case, the system only obtains $1/n$ of the maximum system-wide throughput.

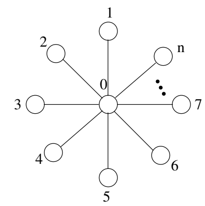

Fig. 3. Flow contention graph in the example of low throughput NEs.

Above, we have obtained a number of results on NEs. In particular, we see that in some cases some NEs can be Pareto-suboptimal or result in low system-wide throughput. If we let the system evolve by itself, the system may converge to a NE that is not desirable. To solve this problem, we propose to design an incentive-compatible channel assignment protocol that can achieve maximal throughput and Pareto optimality.

IV. Protocol for Maximum System-Wide Throughput and Pareto Optimality

In different games, the NEs may have different properties. In Section III, we show that without incentive-compatible schemes, in the multi-collision-domain non-cooperative channel assignment game, some NEs can be Pareto-suboptimal or result in low system-wide throughput. Our findings raise the need for incentive-compatible channel assignment protocols to achieve NEs with desirable Pareto-optimality and system-wide throughput.

In this section, we design *PMT*, an incentive-compatible channel assignment protocol that guarantees that all the NEs have the maximum system-wide throughput and are Pareto-optimal.

A. The PMT Protocol

Maximizing the system-wide throughput in multiple collision domains is not a trivial task even if all involved players are cooperative ([5], [26]). Given the selfishness of the players, it is even more challenging to ensure that all players use the channels in such a way that the maximum system-wide throughput is achieved. To solve this problem, we use an economic tool, payment, to stimulate players to choose channels cooperatively, so that all the NEs that the system can converge to have the maximum system-wide throughput. In the following, we first introduce *Independent Set* IDs, which play an important role in our protocol. Then, we present the design of our payment function and the entire PMT.

Independent set ID $i._{MISID}$**.** Before the channel assignment game starts, for each player an independent set ID in the flow contention graph is assigned as an input. Denote i.MISID the ID of the independent set that player i belongs to, which can be obtained by running an algorithm for maximal independent sets (MIS). To compute the independent set IDs, we can adopt some approximation algorithms for MIS partition (e.g. [29]), which provide good performance as well as time efficiency. [6] Protocol 1 shows the pseudo code for MIS partition using the algorithm in [29], where $d(i)$ is the degree of the node, and N_i is the set of i's neighbors in the remaining graph.

Protocol 1 MIS: Maximal independent sets partition [29]

INPUT: Interference graph G, with the vertex set $V(G)$.
OUTPUT: $i.{}_{MISID}$, $\forall\ i \in V(G)$.
$MISID = 1$.
$G' \leftarrow G$.
while $G' \neq \Phi$ **do**
  $I \leftarrow G$.
  while $I \neq \Phi$ **do**
    Choose i such that $d(i) = \min_{v \in V(I)} d(v)$.
    $i.{}_{MISID} = MISID$.
    $I \leftarrow I - \{i\} - N_i$.
    $G' \leftarrow G' - \{i\}$.
  end while
  $MISID = MISID + 1$.
end while

Each $i.{}_{MISID}$ is known when our PMT starts. We assume that the maximal independent sets are sorted according to its size, and a smaller independent set ID means a larger size. We also assume that after the MIS partition, the number of maximal independent sets is greater than $\left\lceil \frac{|C|}{K} \right\rceil$, because otherwise it is trivial to have all radios assigned without any interference. Actually, in a same independent set, radios from different players do not interfere with each other. Since each player has K radios, it means that the players in one independent set can utilize all their radios on K channels without interference. If the number of independent sets is less than or equal to $\left\lceil \frac{|C|}{K} \right\rceil$, the total number of channels for all the players without interference is less than or equal to $|C|$. In this case, we can just assign K channels to the players in each independent set. This simple assignment solution will not cause any interference. Hence, in this paper, we mainly focus on the non-trivial case that the number of maximal independent sets greater than $\left\lceil \frac{|C|}{K} \right\rceil$.

Virtual currency As in many existing works (e.g., [30]-[32], among many others), we assume that there is a kind of virtual currency in the system.

There is a system administrator in the network, which can simply be a server connected to the Internet. The system administrator maintains an account for each player. Initially, each player can buy some virtual currency, for example using real money. Whenever a player needs the access to some channels, the system administrator charges him a certain amount of fee and updates his account. If a player does not have enough virtual currency to access the channel, it can always buy some using real money. All transactions are cleared in the system administrator. We believe it is natural to ask the channel users to pay for their use of network resources.

Design of payment function In this paper, we assume that all players have enough budgets to make payments and we leave the consideration of budget balance with a limited budget to our future work. To achieve the maximum system-wide throughput, we need to have as many radios as possible to successfully transmit packets. However, not all players can place all their radios in use at the same time due to interference. The most important part of our PMT protocol is a carefully designed payment function which gives players incentives to use channels in such a way that the system has the maximum throughput. In particular, we consider a special independent set τ, which ranks $\left\lceil \frac{|C|}{K} \right\rceil$ among all the independent sets in the decreasing order of sizes.We call τ the threshold independent set. By our payment function, only the players in independent sets larger than τ are encouraged to employ as many radios as possible into channels. We make the independent set that ranks $\left\lceil \frac{|C|}{K} \right\rceil$ among the other independent sets, because our goal here is to achieve maximum system-wide throughput. In particular, we want the $|C|$ channels to be allocated to as many radios as possible. Since each independent set can use K radios without interference, the $|C|$ channels can be assigned to at most $\left\lceil \frac{|C|}{K} \right\rceil$ independent set. Hence we encourage the $|C|$ channels to be assigned to the top $\left\lceil \frac{|C|}{K} \right\rceil$ largest independent set. In this way, maximum system-wide throughput can be achieved. Compared with other methods of determining threshold independent set, ours can guarantee maximum system-wide throughput.

More precisely, the payment of player i is designed as

$$p_i = \frac{(r - \beta) \cdot k_i \cdot (n_\tau - \epsilon)}{n_{i.{}_{MISID}}}. \tag{3}$$

In Eq. (3), recall that k_i is the number of channels on which player i is transmitting packets and $n_{i.{}_{MISID}}$ is the size of the independent set that i is in. The parameter ϵ is a constant positive number smaller than 1; n_τ denotes the size of the threshold independent set τ. The introduction of ϵ guarantees that when the player is in the threshold independent set, i.e., $n_{i.{}_{MISID}} = n_\tau$, the player is encouraged to use their radios as many as possible (as shown in the utility function later). (Here we assume the threshold independent set is unique. In some cases, there may be more than one threshold independent set of size n_τ in the result of the MIS partition algorithm. If so, the system administrator can arbitrarily choose one of them.)

From this payment formula, we can see that if a player employs more radios (i.e., k_i is larger), its payment

[6] One may notice that computing the MIS is NP-hard. However, because the size of flow contention graph is usually small, it is *practical* to use exponential time algorithms.

is correspondingly higher. Moreover, when the network system can provide better communication services (i.e., $(r-\beta)$ is higher), the players need to pay more. More importantly, in order to control how the nodes employ their radios and thus achieve maximum throughput, in the payment formula we have that nodes in larger independent set can pay less (when $n_{i.MISID}$ is smaller). In this way, we encourage the nodes to employ as many as possible radios at the same time.

Plugging the payment formula into the payoff of each player defined in Section II-B, we can get the following equation.

$$\begin{aligned} u_i &= \sum_{c\in C} r \cdot (S_i^c \cdot \prod_{j\in N_i}(1-S_j^c)) - \beta k_i \\ &\quad - \frac{(r-\beta)\cdot k_i \cdot (n_\tau - \epsilon)}{n_{i.MISID}} \end{aligned}$$

Assuming that there is no collision, the above equation of utility becomes[7]:

$$u_i = (r-\beta)(1-\frac{n_\tau - \epsilon}{n_{i.MISID}})k_i. \quad (4)$$

In the payoff function, we let one unit of throughput, one unit of energy cost, and one unit of payment all equal to one unit of utility when counting the total payoff. This assumption does not affect our analysis of players' payoffs. We can always adjust the coefficient of unit conversion if necessary.

As we can easily see, each player in independent sets larger than n_τ will get higher payoff if he increases his number of radios to transmit packets (because n_τ and $n_{i.MISID}$ are both integers and $0<\epsilon<1$). On the other hand, players with $n_{i.MISID} < n_\tau$ will decrease their payoffs if they use more radios to transmit packets.

PMT protocol. We now provide the pseudo code of our PMT protocol (see Protocol 2), which guarantees that all NEs maximize the system-wide throughput. Our PMT protocol is a distributed protocol with imperfect information, which does not assume that each node has the perfect information about other nodes' channel assignment. In this paper we assume that after running the maximal independent set partition algorithm, the central authority sends the information $n_{i.MISID}$ and n_{tau} to each node in the system, before the nodes can assign their radios to channels. Each node does not need to know who is in its interference set, but he can sense the interference when at least one of his neighbors and him are using the same channel at the same time. It is due to the broadcasting nature of wireless network communications. By trying to assign required number of radios to different channels and avoid interference from a local view (as shown in line 8 and line 11 in PMT protocol), the system will gradually converge to NE with desirable system properties without interference. As long as there is no change in the system topology, there is no need to communicate to each node at every time period.

[7]Note that collisions may occur. We have shown that when collisions occur the system state is not a Nash equilibrium and the players unnecessarily lose payoffs. Our goal is to design protocol which maximizes the throughput and thus here we only focus on the cases when there is no collision for simplification and clarity.

Protocol 2 PMT: Multi-radio channel assignment protocol for maximum system-wide throughput and Pareto optimality

1: **INPUT:** number of radios per player K; the set of available channels C; independent set size $n_{i.MISID}$ for each player i; size of the threshold independent set: n_τ.
2: RandomChannelAssignment();
3: **if** $|C| < n_{max}\cdot K$ **then**
4: **while** there is any change compared with last round **do**
5: **for** each player i **do**
6: **if** backoff counter is 0 **then**
7: **if** ($n_{i.MISID} > n_\tau$ **or** $n_{i.MISID} = n_\tau$ **and** $|C| \mod K = 0$) **and** the number of spare radios (k_i^s) is greater than 0 **then**
8: Assign the all radio(s) to channels such that no interference exists from player i's local view;
9: **end if**
10: **if** $n_{i.MISID} = n_\tau$ **and** $k_i^s > K - (|C| \mod K)$ **then**
11: Assign the spare radio(s) to other channels such that no interference exists from player i's local view, to achieve that $k_i^s = K - (|C| \mod K)$;
12: **end if**
13: **if** $n_{i.MISID} < n_\tau$ **and** $k_i^s < K$ **then**
14: Do not assign any radio to any channel.
15: **end if**
16: Reset the backoff counter to a new value;
17: **else**
18: Decrease the backoff counter value by 1;
19: **end if**
20: **end for**
21: **end while**
22: **end if**

At the beginning of each game, players execute the PMT protocol and keep the obtained channel assignment until some players change their strategies. The PMT protocol is a distributed protocol that works in a round-based fashion. After the initial random assignment, each player tries to change the channel assignment to his radios for better utility. In order to guarantee that there is only one player changing his strategy in one round, we use the mechanism of backoff window (as explained later in this paragraph). The players change the channel assignment in the following way. i in independent sets larger than n_τ checks whether all his radios are successfully transmitting packets (i.e. the number of spare radios (k_i^s)is 0). If not, he assigns the spare radio(s) to other channels in order to improve his total rate (line 5-7). Here, by a spare radio we mean a radio that is not successfully transmitting packets. Each player in the threshold independent set will stop changing his channel assignment once he has $|C|\ mod\ K$ radios successfully transmitting packets (line 8-9). For players in independent sets smaller than n_τ, not using any radio is the best strategy. We implement the backoff window as follows. Each player randomly chooses an initial value for his backoff counter from $\{1, 2, \cdots, W\}$, where W is the size of back-off window, with uniform probability. In this way, the backoff counter of each player is different from that of any other player. Since the backoff counter only decreases by 1 in one round, there is only one backoff counter becomes 0 each time PMT runs. Therefore in one round, there is at most one player who changes his strategy.

Now we take the system shown in Fig. 2 as an example input to protocol PMT and see how PMT runs. With the example shown in Figure 2, there are two channels available and each player has 2 radios. There are two independent set in the system: $\{1, 2, \cdots, n\}$ and $\{0\}$, The threshold independent set ranks $\left\lceil \frac{|C|}{K} \right\rceil$ in terms of size, and hence $n_\tau = n$. With any random initial assignment, for any player in $\{1, 2, \cdots, n\}$, if he has any spare radio(s), it satisfies that $n_{i.MISID} = n_\tau$ and $|C| \mod K = 0$, and line 8 in PMT Protocol will be executed, i.e., he assigns spare radio (s) to the available channels. If he has no spare radios, then there is nothing to change in the channel assignment. For the player 0, it satisfies the conditions in line 13, i.e., $n_{i.MISID} < n_\tau$. He does not assign any radios to any channel. The PMT protocol in this simple example stops after two rounds. Then the output of the protocol PMT is that player 0 does not assign any radio to any channel and all players in $\{1, 2, \cdots, n\}$ assign both radios to the two channels.

B. Analysis of PMT Protocol

1) Incentive Compatibility:

Theorem 2: If PMT is used, all the NEs satisfy that $\forall i$,

$$k_i = \begin{cases} K & \text{if } n_{i.MISID} > n_\tau \\ |C| \mod K & \text{if } n_{i.MISID} = n_\tau \\ 0 & \text{if } n_{i.MISID} < n_\tau \end{cases} \quad (5)$$

Proof: We first show that PMT will reach the state s^* which satisfies (5). Then we show s^* is a Nash equilibrium, i.e., the system converges at s^*.

To show that PMT will always reach the state that satisfies Eq. (5), first, we notice that (5) is achievable within the system capacity. Recall that there are $\left\lceil \frac{|C|}{K} \right\rceil - 1$ independent sets that have sizes greater than n_τ and 1 independent set of size n_τ. When players in the same independent set allocate their radios on the same set of channels, the total number of channels without interference required by (5) is $(\left\lceil \frac{|C|}{K} \right\rceil - 1)K + |C| \ mod \ K$, which is exactly the number of channels in the system $|C|$. On the other hand, the system will not stabilize in any state that does not satisfy (5), due to the strategy changing conditions in PMT (line 5 and 8). In each round of PMT we only have one player who changes his assignment so that oscillation of strategy changes can be avoided. Hence there is no possible state in the system that has 0 probability to lead to the state satisfying Eq. (5). Therefore, with PMT, the system will always reach the state that satisfies Eq. (5).

Now we show that state s^* which satisfies (5) is a Nash equilibrium, which guarantees that players do not have incentives to deviate from s^* unilaterally.

Let u_i' denote the payoff of i by taking other strategy s_i' that does not satisfy (5). k_i' is used in s_i'. Given $s*_{-i}$, we distinguish two possible types of s_i', i.e., (1) those result in interference and (2) those don't. Since those s_i' that result in interference will clearly bring lower payoffs for player i than those that avoid interference, in the proof, we only consider those s_i' of type (2). If we can prove that even the second type of s_i' cannot increase the payoff of i, then all possible s_i' cannot either.

There are 3 possible cases as follows.

Case 1. $n_{i.MISID} < n_\tau$.

$$u_i' - u_i^* = (r - \beta)(1 - \frac{n_\tau - \epsilon}{n_{i.MISID}})k_i' \leq 0,$$

because $\frac{n_\tau - \epsilon}{n_{i.MISID}} > 1$.

Case 2. $n_{i.MISID} > n_\tau$.

$$u_i' - u_i^* = (r - \beta)(1 - \frac{n_\tau - \epsilon}{n_{i.MISID}})(k_i' - K) \leq 0,$$

since $\frac{n_\tau - \epsilon}{n_{i.MISID}} < 1$ and $k_i' - K \leq 0$.

Case 3. $n_{i.MISID} = n_\tau$. If the players not in the threshold-independent-set keep the channel assignment results as in (5), the number of channels that player i can use without interference is at most $|C| \ mod \ K$ (i.e. $k_i' \leq |C| \ mod \ K$). Also from $\frac{n_\tau - \epsilon}{n_{i.MISID}} < 1$. We can obtain that $u_i'(s_i', s_{-i}^*) - u_i^*(s_i^*, s_{-i}^*) \leq 0$.

Therefore, if PMT is used, all the NEs satisfy (5). ■

We note that from the case 3 in the proof of Theorem 2, the PMT protocol is ex-post incentive compatible. The PMT protocol is not dominant strategy incentive compatible. This is because for the case that $n_{i.MISID} = n_\tau$, i.e., player is in the threshold independent set, the player can obtain higher utility by assigning more radios than $|C| mod K$, when players in larger independent set assign less radios than K.

2) Throughput and Optimality:

Theorem 3: (Throughput Maximization and Pareto Optimality) If the PMT protocol is used, all the NEs achieve the maximum system-wide throughput. Furthermore, all the NEs are Pareto-optimal.

Proof: We denote system-wide throughput as the sum of the throughput in each channel. $\sum_{c \in C} R_c = \sum_{c \in C} k_c r$, where k_c is the number of radios using channel c in the system. In the convergence state of PMT, $\sum_{c \in C} k_c$ can not be increased by other ways of channel assignment, because a) for players in independent sets smaller than or equal to the threshold independent set, it is impossible to put their spare radios on channels that are used by other players in larger independent sets without any interference, since otherwise it will contradict with the definition of maximal independent set, b) for players in independent sets larger than the threshold independent set, they do not have spare radios to increase throughput (see (5)). Hence the system-wide throughput $\sum_{c \in C} R_c = \sum_{c \in C} k_c r$ is maximized.

The NEs that guarantee system-wide maximum throughput are also Pareto-optimal. This can be proved by contradiction. Note that in any NE, the throughput of each player is proportional to its payoff. If it is not Pareto-optimal, it implies that some players can increase their throughputs without decreasing any other's throughput. Consequently, the system-wide throughput can be better off, which contradicts the throughput maximization. ■

Our PMT guarantees that all NEs are Pareto-optimal, which means that the outcomes of the non-cooperative channel assignment achieve social optimality.

When using different maximal independent set partition approximation algorithms, it does not affect the incentive compatibility of our PMT protocol. From the proof of Theorem 2, no matter how maximal independent sets are partitioned, as long as there is a $n_{i.MISID}$ for each player i, our carefully designed payment formula will make sure that PMT protocol is incentive compatible, i.e., the system will always converge to a desirable NE. Different maximal independent set partition approximation algorithms do have effects on achieving system throughput maximization in the system. We would like to note that it is NP-hard to solve the throughput maximization problem for multiple collision domains in general, and different MIS approximation algorithms may well lead to different throughputs in the system. Our PMT algorithm theoretically guarantees that for each system, as long as the maximal independent set partition result is correct, the PMT protocol will produce the maximum system-wide throughput.

3) Fairness Issue: In the PMT protocol, in order to achieve maximum system throughput, the individual throughput of the players in the independent sets smaller than the threshold set is sacrificed. In particular, those players are not assigning any of their radios to any channel. This causes a fairness issue for the system. Here we first theoretically analyze the upper bound of the ratio of such silent players and then we discuss possible solutions for this fairness issue.

Theorem 4: In PMT protocol, the upper bound for the ratio of silent players is $1 - \frac{\lceil \frac{|C|}{K} \rceil}{n}$, where n is the number of maximal independent sets in the system after the maximal independent set partition.

Proof: Since the threshold independent set ranks $\lceil \frac{|C|}{K} \rceil$ in terms of set size, the ratio of players in the threshold set and larger independent set is greater than $\frac{\lceil \frac{|C|}{K} \rceil}{n}$. Hence the ratio of players in the independent set (i.e., silent players) is smaller than the threshold set is smaller than $1 - \frac{\lceil \frac{|C|}{K} \rceil}{n}$. ■

We observe that the silent players are in smaller maximal independent set. It is because compared with other players, they will interfere with more players if using the channels. So in order to achieve high system throughput, these players need to turn off their radios and let more others use the channel resources. However, long term starvation should be avoided in the wireless networks. In order to solve this issue, one possible solution is to periodically re-compute the independent set ID for each player to allow the silent node changing its independent set to a larger one, increasing the probability to have more channel access.

C. Implementation Issues

Our PMT protocol works in wireless systems that have a protocol or mechanism that enables the wireless devices to use multiple channels to communicate at the same time. For example, [28] is one of such muti-radio protocols for IEEE 802.11 wireless networks. Our protocol let the nodes coordinate to achieve a channel allocation of their radios. To perform the PMT protocol, the system administrator sends a message to each node i whose radio needs to be reconfigured, which contains the n_τ and $n_{i_{MISID}}$. After receiving the acknowledgment from each node, the system administrator sends a synchronization message, and it invokes the PMT protocol described in Section IV-A.

The computational overhead of our channel assignment is mainly from two parts, i.e., computing the maximal independent sets and the time required for system convergence. For the first part of overhead, as the system grows larger, further performance optimization is needed, e.g., by utilizing more efficient heuristic algorithm to compute maximal independent sets and by using smaller amount of time in each round of nodes coordination. In Section V-D we will investigate the system convergence time in greater details.

D. Advanced Model and Analysis

With the considerations of more complicated conditions, non-cooperative channel assignment problem in multi-radio multi-channel wireless networks can be modeled in more advanced game model. For example, in each round of channel assignment, each node can observe the action of his neighbors (by sensing the interference), and accordingly change his own action in the next round of channel assignment, to avoid interference. This sequential nature can be modeled by a sequential game (or dynamic game) in the extensive form. In this model, our payment scheme needs to be extended for each possible action in the sequential game, so that a subgame Nash equilibrium can be achieved. Here a subgame Nash equilibrium is a solution concept in dynamic games, which guarantees a Nash equilibrium for every subgame of the original dynamic game.

V. EVALUATIONS

In this section, we carry out a number of experiments in GloMoSim [16] to verify the effectiveness of PMT. In the implementation, we use the approximation algorithm in [29] to compute the maximal independent set before the game starts.

We first generate a network of 20 pairs of nodes with a random topology in a $1000 \times 1000\text{m}^2$ region. In each pair, the two nodes are 20 meters away from each other. The flow contention graph is shown in Fig. 4. There is a bidirectional single-hop flow between the two nodes in each pair at a constant bit rate, and we vary the traffic demand rate in the experiments.

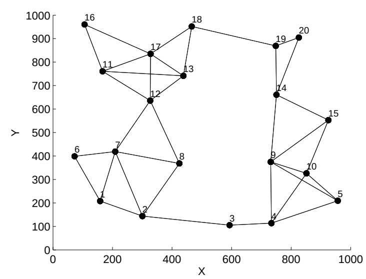

Fig. 4. The flow contention graph of the network.

We test our protocols with two sets of parameters. In one set, we let the number of channels $|C| = 12$, channel capacity $R = 54$ Mbps, and the number of radios $K = 4$. In the other set, we let $|C| = 3$, $R = 11$ Mbps and $K = 2$. Each result is obtained by averaging over 500 runs. We set $r = 2$, $\beta = 0.25$, and $\epsilon = 0.1$.

In section V-A we evaluate the payment for each node in the system in two different settings. In Section V-B, we evaluate the effectiveness of PMT in achieving maximum system throughput. In particular, we measure the system throughput in the stable states when PMT is used, and compare the results with the situation when no incentive-compatible protocol is used. In Section V-C, we study the fairness property in the system when running PMT. Furthermore, in Section V-D we investigate the system convergence process, and find that the protocol can make the system converge to the stable state fairly quickly. We evaluate the system efficiency for PMT in Section V-E. The experiments in the above subsections are performed using the system topology shown in Fig. 4, in Section V-F, we randomly place the 20 nodes in the $1000 \times 1000\text{m}^2$ region, and analyze the average results of throughput and fairness for different system topologies.

A. Payment

In this section, we closely observe the payment that each node makes in our PMT protocol, in two different settings. After running the maximal independent set partition algorithm, we find that when $|C| = 12$ and $K = 4$, the size of threshold independent set $n_\tau = 4$; when $|C| = 3$ and $K = 2$, we have $n_\tau = 6$.

Fig. 5 plots the payment of each node in the system, when $|C| = 12$ and $K = 4$. We notice that node 9 and node 12 are making 0 payments. It is because they are not using any channels and correspondingly, they do not need pay anything. Similar results for the setting that $|C| = 12$ and $K = 4$ are shown in Fig. 6. We observe that when the system setting changes, the threshold independent set and the number of radios that each node uses may correspondingly change. Consequently, the payment of the same node is different for different system settings.

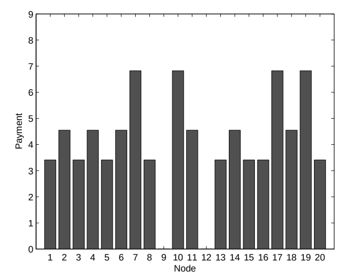

Fig. 5. Payment of each node in the system. ($|C| = 12$, $K = 4$, $R = 54Mbps$.)

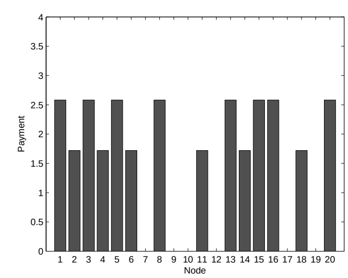

Fig. 6. Payment of each node in the system. ($|C| = 3$, $K = 2$, $R = 54Mbps$.)

B. Evaluation of Throughput

We measure the system throughputs of PMT, as well as the average system throughput of random NEs (which will be reached when the system has no incentive-compatible protocol). Our objective is to design a channel assignment protocol such that the channel assignment which leads to maximal throughput also meets the interest of each player. In this paper, as for the degree of incentive compatibility, we use the notion Nash equilibrium, and thus by incentive-compatible protocol, we mean that the protocol by which the maximum throughput channel assignment is the Nash equilibrium strategy for each player. Therefore, from Theorem 2, we know that PMT is an incentive-compatible protocol. We note that the NE convergence algorithm in [8] do not use any incentive-compatible scheme to influence the NEs that the system will converge to, and hence by using the algorithm in [8], the system can converge to any random NE. Thus the comparison results shown in Section V-B and Section V-C are actually the comparison between PMT and [8]. The results of system throughput are illustrated in Figs. 7 and 8.

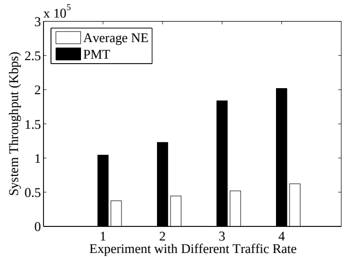

Fig. 7. Aggregate system throughput. ($|C| = 3$, $K = 2$, $R = 11Mbps$. The traffic demand rates of experiment $1, 2, 3, 4$ are $8Mbps$, $10Mbps$, $16Mbps$ and $20Mbps$ respectively.)

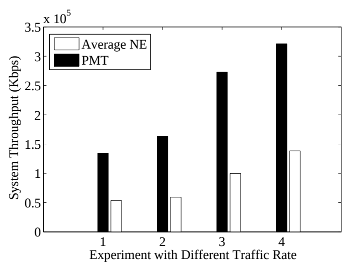

Fig. 8. Aggregate system throughput. ($|C| = 12$, $K = 4$, $R = 54Mbps$. The traffic demand rates of experiment $1, 2, 3, 4$ are $8Mbps$, $10Mbps$, $16Mbps$ and $20Mbps$ respectively.)

In Fig. 7, we show the system throughputs of 4 different experiments with different traffic demand rates, when we set $|C| = 3$, $K = 2$ and $R = 11$ $Mbps$. As we can see, for each experiment, the system-wide throughput achieved by PMT is much higher than the average of random NEs with no incentive-compatible protocols. It implies that, compared with the systems without incentive-compatible channel assignment protocols, PMT greatly improves the system-wide throughput.

Similar conclusions can be drawn from Fig. 8, when we set $|C| = 12$, $K = 4$ and $R = 54Mbps$.

C. Fairness

Now we examine the fairness property of our PMT protocol. As in the design of PMT, our objective is to maximize the throughput. Then it is important to make sure that the throughput maximization does not sacrifice too much fairness in the system. To this end, we measure the fairness in terms of individual throughput. We utilize the Jain's fairness index [33] as a quantitative metric. Fairness index is a real number, ranging from 0.05(worst) to 1(best) for the system of 20 players. We measure the fairness indices of the system's stable states achieved by the PMT, and also the average fairness indices of random NEs, which are reached when there is no incentive-compatible channel assignment protocol. We repeat the experiments with different traffic rates and in two different settings (Setting 1: $|C| = 12$ $K = 4$ $R = 54Mbps$; Setting 2: $|C| = 3$ $K = 2$ $R = 11Mbps$). The results are shown in Figure 9. In the figure, we can see that When $|C| = 12$ and $K = 4$, the average fairness index of random NEs is better than that achieved by the PMT. But there is little difference between the fairness indices achieved by the PMT and the average fairness indices of random NEs. This suggests that the PMT has less fairness loss when the traffic demand is closer to the system capacity.

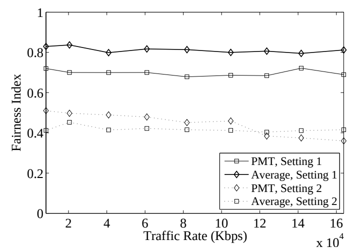

Fig. 9. Fairness index of PMT.

In the experiments, we found that in setting of $|C| = 12$ and $K = 4$, there are 2 silent nodes in the system and when the channel resource becomes more limited, i.e., $|C| = 3$ and $K = 2$, there are more silent nodes in the network, 6 in total.

D. System Convergence

The results stated above are on the system performance in the stable states. In this subsection, our goal is to examine the process the system converges to the stable states. We take records of the system-wide throughput for PMT when the systems are evolving, and show them in Fig. 10. The traffic demand rate is set to 80 Mbps in this experiment. We can see that PMT converges in 0.5 seconds. Therefore, PMT can successfully make the system converge to the stable states and the convergence is very fast.

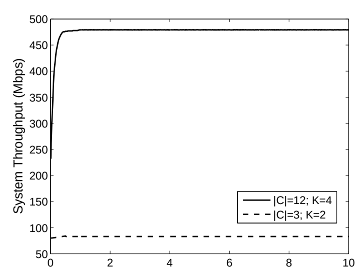

Fig. 10. Convergence of system throughput of PMT.

E. Economic Efficiency

In this subsection, we study the economic efficiency of the system using our PMT protocol. We use an *efficiency ratio* to characterize the efficiency of the system. In particular, the efficiency ratio is defined as the ratio between the sum of payoffs of all players in the Pareto-optimal solution and the sum of payoffs by the current strategy profile. We present an example run of PMT protocol for 10s in Figure 11 in the setting of $|C| = 12, K = 4$. We can observe that PMT protocol quickly converges to a Nash equilibrium. When the system is stable, the efficiency ratio stays at the value of 1. Hence we conclude that our PMT protocol makes the system converge to a state with high efficiency.

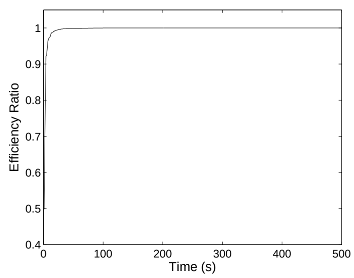

Fig. 11. Convergence of system efficiency ratio of PMT.

F. Results over Different System Topologies

In this subsection, we change the system topology and evaluate the system throughput and fairness in different network topologies when PMT is used. In particular, we generate 10 network topologies of 20 nodes. In each topology, we randomly place the 20 nodes in the $1000 \times 1000\text{m}^2$ region and make sure that the maximum degree of nodes is no more than 6. We measure the system throughput and fairness for each network topology and show the average results and standard deviation in Fig. 12 and Fig. 13 respectively. Fig. 12 and Fig. 13 demonstrate that our PMT protocol works well for different network topologies and the performance is stable.

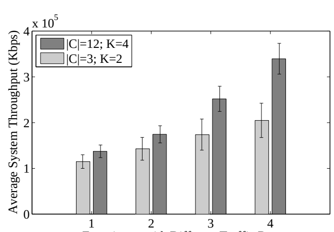

Fig. 12. Average system throughput of PMT in 10 randomly generated network topologies.

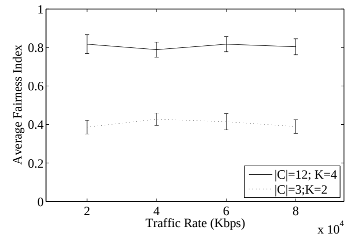

Fig. 13. Average fairness index of PMT in 10 randomly generated network topologies.

VI. Related Work

In WMNs, channel assignment problem has been considered for multi-radio devices [5]–[7], [28], [34]–[36]. It is important because simultaneously transmitting packets with multiple radios on orthogonal channels can significantly increase the system capacity. Both centralized algorithms [6] and distributed protocols [7] have been developed for multi-radio channel assignment for WMNs. Alicherry *et al.* [5] jointly consider channel assignment and routing to optimize the system throughput.

All these channel assignment protocols above are under the same assumption that devices in the network are cooperative in that they never deviate from the protocol. As devices can be selfish when accessing the channels, recently researchers begin to study the non-cooperative channel assignment problem [37]–[39], especially for cognitive radio networks. For example, Nie et. al. [37] propose a dynamic spectrum allocation scheme based on a potential game model and introduce some learning algorithms for different payoff functions. Thomas *et al.* [38] also utilize a potential game model to study how to minimize transmission power while maintaining connectivity by channel assignment. The major difference between these works and ours is that they assume that the selfish player has only a single radio, while we study the non-cooperative, multi-radio channel assignment problem in which selfish devices have multiple radios to manipulate. Since the players and their objectives (i.e., payoff functions) in the game are significantly different, their solutions can not be applied in the more general case that we are focusing on in this paper.

For wireless networks in which devices have multiple radios, Felegyhazi *et al.* [8] are the first to introduce the strategic game model to study the non-cooperative channel assignment problem. They study the NEs in this game and find out that despite of non-cooperative behavior of the players, the NEs result in load balancing. The differences between our work and [8] are in three aspects: First, in [8], only the scenario that the all transmitting nodes are in a single collision clique is considered, while in our papers, we consider the non-cooperative channel assignment in more general and complicated cases of the system topology, i.e., it contains multiple collision domains. The elegant results of load balancing by Felegyhazi *et al.* for single collision domains is based on the fact that each pair of nodes in the system will interfere each other and thus cannot be applied to multiple collision domains. Second, in our work, our goal is to design incentive-compatible channel assignment protocols which can achieve maximum system throughput and Pareto-optimality. In [8], the maximum system throughput is not guaranteed by their non-cooperative channel assignment algorithms. Third, in [8], central and distributed algorithms are designed for system convergence to Nash equilibrium, but no mechanism design approach is used to influence the convergence. In this paper, we use payment based approach to make sure that the Nash equilibrium that the system will converge to is a desirable one with maximum system throughput and Pareto-optimality. In a later work, Wu *et al.* [9], [10] propose a stronger solution for this game which is strictly dominant and extend the model such that players can have different number of radios. Gao *et al.* [11] go one step further to consider the non-cooperative channel allocation for multi-hop wireless networks. However, all the three works (including [11]) assume that all the nodes in the network are within a single collision domain. In this paper we remove this assumption and study the non-cooperative, multi-radio channel assignment problem in multiple collision domains, with a focus on system throughput.

Recently, Vallam, Kanagasabapathy and Murthy [12] also studied the problem of non-cooperative channel assignment in multi-channel multi-radio networks with multiple collision domains. They have obtained nice and solid results. In particular, a new fairness measure in multiple collision domains is proposed and fair equilibrium conditions are derived. Based on the conditions, three nice channel assignment algorithms are also proposed. In fact, they are the *first* to study the problem of non-cooperative channel assignment in multi-channel multi-radio networks with multiple collision domains. The differences between [12] and our work are two-fold. First, for system performance, our main focus is on throughput, while [12] has more significant contributions in system fairness. Second, in our paper, we use a payment-based approach to achieve players' incentive-compatibility with the objective of maximum system throughput, while [12] leverages advanced learning algorithm in the system convergence.

VII. Conclusion

In this paper, we have systematically studied the problem of non-cooperative, multi-radio channel assignment in multiple collision domains, and obtained quantified results on economic efficiency, and throughput. Our results show that, without an incentive-compatible channel assignment protocol, the system is likely to converge to NEs with undesired properties like low throughput and Pareto suboptimality. To avoid this, we propose an incentive-compatible protocol for multi-radio channel assignment in multiple collision domains. This protocol guarantees that the system converges to NEs that have the maximum throughput and Pareto optimality.

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